It can be shown that when the entire base is submerged in the snow the same equation applies with $b' = b$.

We assume that the snow force acts perpendicular to the base and varies linearly across the base. This distributed snow force can be represented by a concentrated snow force that acts at point E at a distance a from point D as shown in Figure E.2. When the entire segment is buried in the snow the distance between points D and E is

$$a = \frac{b}{6}\left(\frac{u_{\max} - u_{\min}}{u_{\max} + u_{\min}}\right) + x_c \qquad \text{(fully submerged)} \qquad \text{(E.4)}$$

where $u_{\min}$ is the depth of the shallow edge in the snow, and x_c is the distance from the center of the element to the centroid.

$$a = \frac{b}{2} - \frac{u_{\max}}{3\sin\beta} + x_c \qquad \text{(partially submerged)} \qquad \text{(E.5)}$$

The force (per unit length) f^{snow} acting at point E is equivalent to a force f^{snow} and a torque (per unit length) t^{snow} acting at point D where

$$t^{snow} = af^{snow} \qquad \text{(E.6)}$$

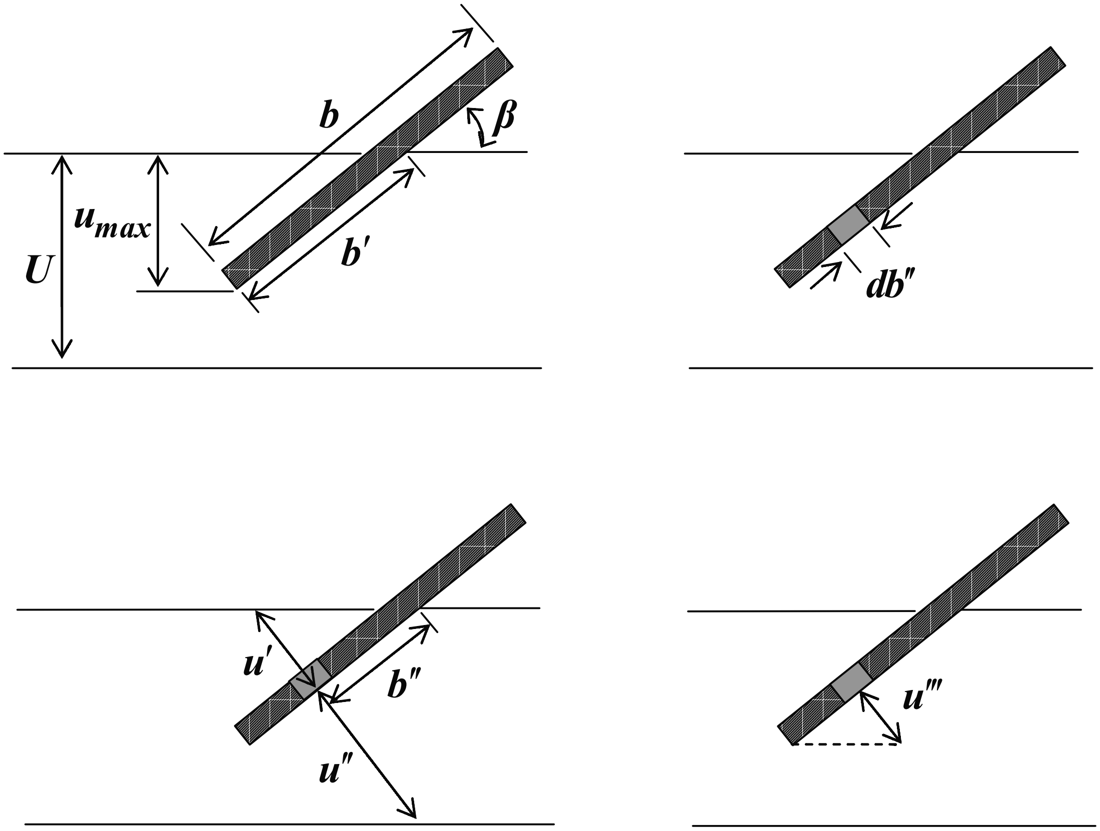

Figure E.1: The partially submerged snowboard.

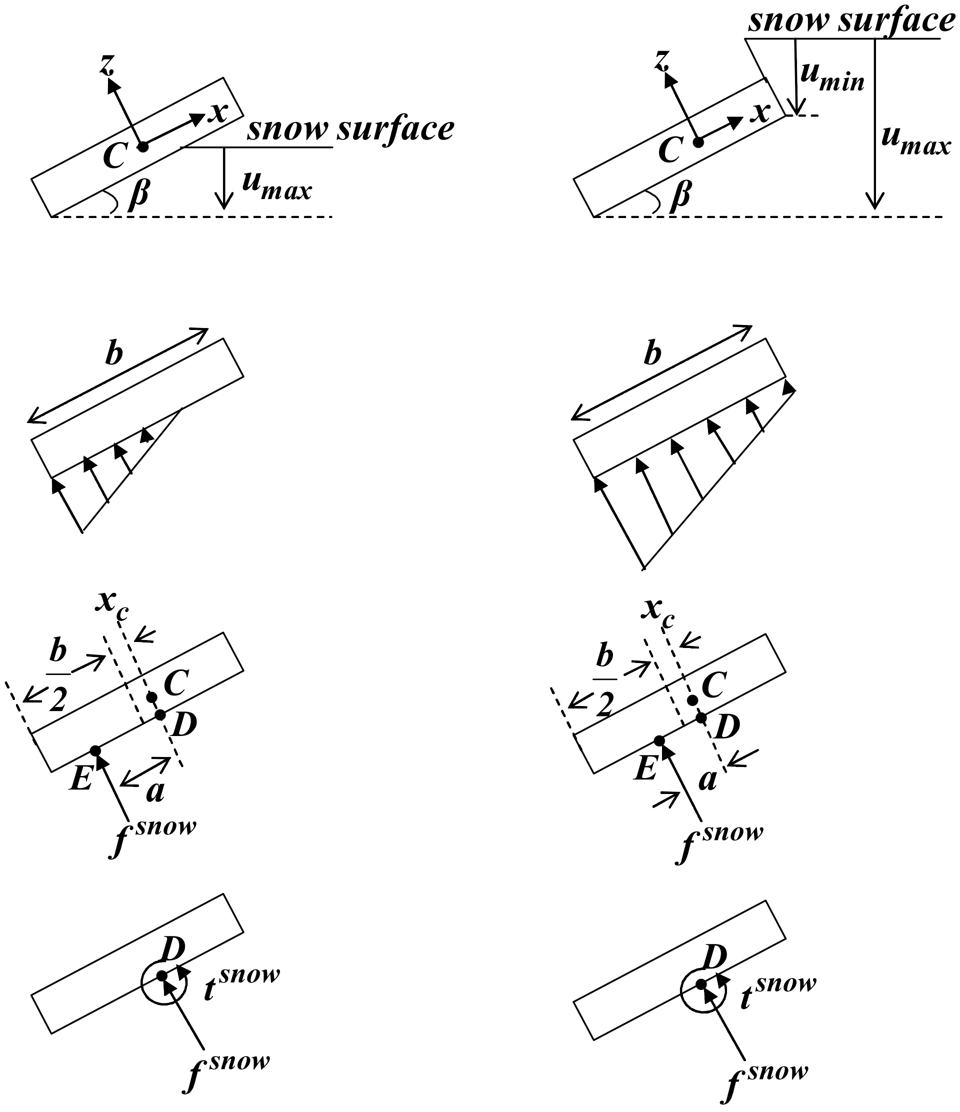

Figure E.2: The snow force and torque.

Appendix F

Method of Solution

F.1 Representation of the Snowboard

We divide the snowboard into $n-1$ elements. The elements are connected at nodes which are numbered such that node 1 is at the tail, node nb is just aft of the back boot, node nf is just forward of the front boot, and node n is at the tip (Figure F.1). The nodes are located at the centroid of each cross section. The $\widetilde{X}-\widetilde{Y}-\widetilde{Z}$ coordinate system is fixed to the snowboard (Figure F.2). The origin is positioned at the mid-running surface MRS, the midpoint between the forward and the aft contact points. It is centered at the midpoint between the two edges and fixed to the base of the snowboard. The location of each node of the snowboard is given in this snowboard coordinate system. The coordinate of the *i*th node being represented by $\widetilde{X}_i$, $\widetilde{Y}_i$, $\widetilde{Z}_i$ as shown in (Figure F.3).

The asymmetry of the snowboard is characterized by the offset. The offset angle λ_i is calculated as follows (Figure F.4). For the undeformed snowboard we can determine the angle ζ_i at the *i*th node which represents the angle between the *i*th segment and the $\widetilde{Y}$ axis in the $\widetilde{X}-\widetilde{Y}$ plane to be

$$\zeta_i = -\arctan\left(\frac{\widetilde{X}_{i+1}-\widetilde{X}_i}{\widetilde{Y}_{i+1}-\widetilde{Y}_i}\right) \tag{F.1}$$

Knowing the values of ζ_i for at nodes 1 to *n*-1 we can determine the relative angle between two adjacent segments. Thus, the offset angle λ at nodes 2 to *n*-1 is defined as

$$\lambda_i = \zeta_i - \zeta_{i-1} \tag{F.2}$$

Similarly, we can determine the bending angle due to camber for the undeformed snowboard. The angle between the *i*th segment and the $\widetilde{Y}$ axis in the $\widetilde{Y}-\widetilde{Z}$ plane as

$$\chi_i = \arctan\left(\frac{\widetilde{Z}_{i+1}-\widetilde{Z}_i}{\widetilde{Y}_{i+1}-\widetilde{Y}_i}\right) \tag{F.3}$$

Knowing the values of χ_i for at nodes 1 to *n*-1 we can determine the relative angle between two adjacent segments. Thus, the camber angle θ_i^c at nodes 2 to *n*-1 (Figure F.5), assuming small angles, is defined as

$$\theta_i^c = \chi_i - \chi_{i-1} \tag{F.4}$$

We assume no twist along the length for the undeformed snowboard. The length of the rigid bar is

$$l_i = \sqrt{\left(\widetilde{X}_{i+1}-\widetilde{X}_i\right)^2 + \left(\widetilde{Y}_{i+1}-\widetilde{Y}_i\right)^2 + \left(\widetilde{Z}_{i+1}-\widetilde{Z}_i\right)^2} \tag{F.5}$$

F.2 Determining the Deformed Shape and the Orientation of the Snowboard

We employ the "moment area method" [13, page 531] and concentrate the deformations at the nodes. Thus, each element of the snowboard is represented by a rigid bar connected to the adjacent bars by springs (Figure F.6).

We previously introduced (Appendix A) the $p-q-r$ coordinate system which is attached to the path. The relationship between the "global" $p-q-r$ coordinate system and the "snowboard-fixed" $X-Y-Z$ coordinate system is given by the pitch η, roll β, and yaw α angles as described in Appendix A. The origin of the $X-Y-Z$ coordinate system coordinate system is at point *B*, the midpoint between the *nf* and *nb* nodes. In addition, we have defined a "local" $x-y-z$ coordinate system for each element. If we consider the *i*th element, the origin of this local coordinate system is located at the point *e,* the midpoint between the *i*th and *i*+1 node as illustrated in (Figure F.7). The *y* axis lies along the line connecting the centroids of the *i*th and *i*+1 nodes. The *z* axis is perpendicular to the *g-h-d* plane. The *x* axis is normal to the $y-z$ plane. The $X-Y-Z$ coordinate system is attached to the snowboard such that the Y axis is aligned with the line $a-a$ passing through the nodes *nf* and *nb* (Figure F.8). The Z axis is defined as follows. When the local x axes of the two boot elements are parallel $\left(x_{nb}=x_{nf-1}\right)$, Z is perpendicular to the $x_{nb}-Y$ plane. If x_{nb} and x_{nf-1} are not parallel we define a vector $x^{avg}=x_{nb}+x_{nf-1}$ and Z is perpendicular to the $x^{avg}-Y$ plane. The X axis completes the orthogonal system such that $X=Y\times Z$. We can relate this "snowboard-fixed" $X-Y-Z$ coordinate system to the $\widetilde{X}-\widetilde{Y}-\widetilde{Z}$ coordinate system as shown in (Figure F.9).

To evaluate the spring constants k_i^{EI} and k_i^{GJ} we assume uniform bending and torsional stiffnesses respectively, and, when no bend-twist coupling is present, we write

$$k_i^{EI}=\left[\left(P_{22}\right)_i\left(\frac{1}{\frac{l_{i-1}+l_i}{2}}\right)\right]\qquad k_i^{GJ}=\left[\left(P_{44}\right)_i\left(\frac{1}{\frac{l_{i-1}+l_i}{2}}\right)\right]\tag{F.6}$$

where $\left(P_{22}\right)_i$ and $\left(P_{44}\right)_i$ are the components of the stiffness matrix $[P]=[W]^{-1}$ (Appendix C) as defined at the *i*th node. When we must also account for the bend-twist coupling we define the coupling stiffness k_i^C as follows

$$k_i^{C}=\left[\left(P_{24}\right)_i\left(\frac{1}{\frac{l_{i-1}+l_i}{2}}\right)\right]\tag{F.7}$$

where $\left(P_{24}\right)_i$ is the component of the stiffness matrix which corresponds to bend-twist coupling.

The position of any point along the snowboard may be calculated given the absolute position of each element. Thus, to determine the deformed shape of the snowboard we must prescribe the following: the offset angle λ_i, which represents the rotation about the local *z* axis, the total bending angle θ_i^t, and the twist angle γ_i^t at each interior node (i.e. nodes 2 to *n*-1). The calculation of the offset angle λ_i is defined in

Section F.1 (Figure F.4). The total bending angle θ_i^t consists of two components $\theta_i^t = \theta_i^c + \theta_i^b$ and represents the rotation about the local x axis as illustrated in Figure F.10, where θ_i^t is the total rotation, θ_i^c is the rotation due to camber, and θ_i^b is the rotation due to bending. We further define the twist angle γ_i^t about the local y axis (Figure F.11).

The determination of the position of the deformed snowboard (i.e. the position of the elements) in the global $p-q-r$ coordinate system is performed in two stages. In Stage 1, the position of the snowboard is incrementally determined by calculating the position of the nodes aft of node *i* based on the angles λ_i, θ_i^t, γ_i^t at the *i*th node. In Stage 2, a series of rigid body rotations and a rigid body translation orient the snowboard is such that the snowboard–fixed $X-Y-Z$ coordinate system is related to the global $p-q-r$ coordinate system by the pitch η, roll β, and yaw α angles.

Stage 1 **Step 1** The tail of the snowboard (element 1) is aligned with the global $p-q-r$ coordinate system as illustrated in Figure F.12(a) such that

$$\begin{Bmatrix} r_1 \\ p_1 \\ q_1 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \qquad \begin{Bmatrix} r_2 \\ p_2 \\ q_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ l_1 \\ 0 \end{Bmatrix} \tag{F.8}$$

Step 2 We then perform a rigid body translation to place node 2 at the origin of the $p-q-r$ coordinate system (Figure F.12b) such that

$$\begin{Bmatrix} r_1 \\ p_1 \\ q_1 \end{Bmatrix} = \begin{Bmatrix} 0 \\ -l_1 \\ 0 \end{Bmatrix} \qquad \begin{Bmatrix} r_2 \\ p_2 \\ q_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \tag{F.9}$$

We perform a rigid body rotation to align element 2 with the $p-q-r$ coordinate system. We assume that the rotations occur with respect to the $p-q-r$ coordinate system because the $x-y-z$ coordinates of element 2 are aligned with this frame. The transformation matrices describing the orientation of the local $x-y-z$ coordinate system of element 2 to the global $p-q-r$ coordinate system are defined as

$$T_r = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_2^t & -\sin\theta_2^t \\ 0 & \sin\theta_2^t & \cos\theta_2^t \end{bmatrix} \quad T_p = \begin{bmatrix} \cos\gamma_2^t & 0 & \sin\gamma_2^t \\ 0 & 1 & 0 \\ -\sin\gamma_2^t & 0 & \cos\gamma_2^t \end{bmatrix} \quad T_q = \begin{bmatrix} \cos\lambda_2 & -\sin\lambda_2 & 0 \\ \sin\lambda_2 & \cos\lambda_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{F.10}$$

Therefore, we can determine the position of each node (Figure F.12c) following the rigid body translation to align element 2 with the global coordinate system such that

$$\begin{bmatrix} r_1 \\ p_1 \\ q_1 \end{bmatrix} = \left[T_q T_p T_r\right]^{-1} \begin{bmatrix} 0 \\ -l_1 \\ 0 \end{bmatrix} \tag{F.11}$$

Step 3 Element 2 is now aligned with the global $p-q-r$ coordinate system such that

$$\begin{Bmatrix} r_3 \\ p_3 \\ q_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ l_2 \\ 0 \end{Bmatrix} \tag{F.12}$$

We then perform a rigid body translation to place node 3 at the origin of the $p-q-r$ coordinate system (Figure F.12d), thus the position of nodes aft of node 3 is

$$\begin{Bmatrix} r_j \\ p_j \\ q_j \end{Bmatrix}^{new} = \begin{Bmatrix} r_j \\ p_j \\ q_j \end{Bmatrix}^{old} - \begin{Bmatrix} 0 \\ l_2 \\ 0 \end{Bmatrix} \qquad j = 1,2,3 \tag{F.13}$$

We perform a rigid body rotation to align element 3 with the $p-q-r$ coordinate system (Figure F.12e). We assume that the rotations occur with respect to the $p-q-r$ coordinate system because the $x-y-z$ coordinates of element 3 are aligned with this frame. The transformation matrices describing the orientation of the local $x-y-z$ coordinate system of element 3 to the global $p-q-r$ coordinate system are defined as

$$T_r = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_3^t & -\sin\theta_3^t \\ 0 & \sin\theta_3^t & \cos\theta_3^t \end{bmatrix} \quad T_p = \begin{bmatrix} \cos\gamma_3^t & 0 & \sin\gamma_3^t \\ 0 & 1 & 0 \\ -\sin\gamma_3^t & 0 & \cos\gamma_3^t \end{bmatrix} \quad T_q = \begin{bmatrix} \cos\lambda_3 & -\sin\lambda_3 & 0 \\ \sin\lambda_3 & \cos\lambda_3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{F.14}$$

Therefore, we can determine the position of each node following the rigid body translation to align element 3 with the global coordinate system such that

$$\begin{bmatrix} r_1 & r_2 \\ p_1 & p_2 \\ q_1 & q_2 \end{bmatrix}^{new} = \left[T_q T_p T_r\right]^{-1} \begin{bmatrix} r_1 & r_2 \\ p_1 & p_2 \\ q_1 & q_2 \end{bmatrix}^{old} \tag{F.15}$$

This process is repeated until the position of every node is prescribed and node *n* is located at the origin of the global $p-q-r$ coordinate system (Step *n*).

Stage 2

A rigid body translation and a series of rigid body rotations are performed to align the snowboard fixed origin of the $p-q-r$ coordinate system with point *B* (the midpoint between the two boots, nodes nf and nb) and orient the board such that the *a-a* line corresponds to the p axis. The rigid body translation can be expressed as:

$$\begin{Bmatrix} r_i \\ p_i \\ q_i \end{Bmatrix}^{new} = \begin{Bmatrix} r_i \\ p_i \\ qi \end{Bmatrix}^{old} - \tfrac{1}{2} \begin{Bmatrix} r_{nf} - r_{nb} \\ p_{nf} - p_{nb} \\ q_{nf} - q_{nb} \end{Bmatrix}^{old} \tag{F.16}$$

where the nf and nb subscripts refer to nodes associated with the front boot and back boot respectively. Similarly, two rigid body rotations can be performed to align the *a-a* line with the *p* axis. Initially, the angle between the *a-a* line and the *p* axis in the $p-q$ plane (Figure F.13) is

$$\Psi = \arctan\left(\frac{q_{nf}}{p_{nf}}\right) \tag{F.17}$$

where p_{nf} and q_{nf} are the coordinates of node n_f for a snowboard with point *B* at the origin of the $p-q-r$ coordinate system. Thus, the snowboard is rotated an angle $-\Psi$ about the *r* axis. The new position after this rotation is

$$\begin{Bmatrix} r_i \\ p_i \\ q_i \end{Bmatrix}^{new} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(-\Psi) & -\sin(-\Psi) \\ 0 & \sin(-\Psi) & \cos(-\Psi) \end{bmatrix} \begin{Bmatrix} r_i \\ p_i \\ q_i \end{Bmatrix}^{old} \tag{F.18}$$

Now the angle between the *a-a* line and the p axis in the $p-r$ plane is

$$\Phi = -\arctan\left(\frac{r_{nf}}{p_{nf}}\right) \tag{F.19}$$

where p_{nf} and r_{nf} are the coordinates of node nf following the rotation about the r axis. To orient the *a-a* line with the p axis, the snowboard is rotated $-\Phi$ about the q axis. The new position after this rotation is

$$\begin{Bmatrix} r_i \\ p_i \\ q_i \end{Bmatrix}^{new} = \begin{bmatrix} \cos(-\Phi) & -\sin(-\Phi) & 0 \\ \sin(-\Phi) & \cos(-\Phi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} r_i \\ p_i \\ q_i \end{Bmatrix}^{old} \tag{F.20}$$

We define the X and Z axes a discussed previously. Once we have defined the Z axis $\left(Z = \begin{Bmatrix} r^Z & p^Z & q^Z \end{Bmatrix}\right)$, we can perform one final rigid body rotation of the snowboard to align the Z axis with the q axis in the $q-r$ plane (Figure F.14). The angle between the Z and the q axes in the $q-r$ plane is

$$\Lambda = \arctan\left(\frac{r^Z}{q^Z}\right) \tag{F.21}$$

where r^Z and q^Z are the coordinates of the Z axis in the $p-q-r$ coordinate system. To align Z with the q axis, the snowboard is rotated $-\Lambda$ about the p axis. The new positions of the nodes after this rotation are

$$\begin{Bmatrix} r_i \\ p_i \\ q_i \end{Bmatrix}^{new} = \begin{bmatrix} \cos(-\Lambda) & 0 & \sin(-\Lambda) \\ 0 & 1 & 0 \\ -\sin(-\Lambda) & 0 & \cos(-\Lambda) \end{bmatrix} \begin{Bmatrix} r_i \\ p_i \\ q_i \end{Bmatrix}^{old} \tag{F.22}$$

Following this rotation, the snowboard-fixed $X-Y-Z$ coordinate system is aligned with the global $p-q-r$ coordinate system. To determine the position of the snowboard in the global coordinate system given values of pitch η, roll β, and yaw α we follow the procedure outlined in Appendix A.

It will also be useful to have a set of unit vectors to describe the local coordinate system for each node. We denote the unit vectors pointing in the *x*, *y*, *z* directions at the *i*th element by $\hat{x}_i^{ele}$, $\hat{y}_i^{ele}$, and $\hat{z}_i^{ele}$. The unit vectors describing the local coordinate system of node *i* are defined as follows

$$\begin{gathered} x_i^a = \left(\hat{x}_{i-1}^{ele} + \hat{x}_i^{ele}\right) \quad e_i = \left(\hat{y}_{i-1}^{ele} + \hat{y}_i^{ele}\right) \quad z_i^a = x_i^a \times e_i \\ \hat{x}_i^{node} = \frac{x_i^a}{\left|x_i^a\right|} \quad \hat{z}_i^{node} = \frac{z_i^a}{\left|z_i^a\right|} \quad \hat{y}_i^{node} = \hat{z}_i^{node} \times \hat{x}_i^{node} \end{gathered} \tag{F.23}$$

At nodes 1 and *n* the local unit vectors are defined such that

$$\begin{gathered} \hat{x}_1^{node} = \hat{x}_1^{ele} \quad \hat{y}_1^{node} = \hat{y}_1^{ele} \quad \hat{z}_1^{node} = \hat{z}_1^{ele} \\ \hat{x}_n^{node} = \hat{x}_n^{ele} \quad \hat{y}_n^{node} = \hat{y}_n^{ele} \quad \hat{z}_n^{node} = \hat{z}_n^{ele} \end{gathered} \tag{F.24}$$

We can define the transformation matrix relating the local $x-y-z$ coordinates of the *i*th element to the global $p-q-r$ coordinate system as follows [14, page 321]

$$\left[T_{LG}\right]_i^{ele} = \left[\left\{\hat{x}_i^{ele}\right\} \quad \left\{\hat{y}_i^{ele}\right\} \quad \left\{\hat{z}_i^{ele}\right\}\right] \tag{F.25}$$

Similarly we can define the transformation matrix relating the local $x-y-z$ coordinates of the *i*th node to the global $p-q-r$ coordinate system as follows

$$\left[T_{LG}\right]_i^{node} = \left[\left\{\hat{x}_i^{node}\right\} \quad \left\{\hat{y}_i^{node}\right\} \quad \left\{\hat{z}_i^{node}\right\}\right] \tag{F.26}$$

F.3 Snow Forces

In Appendix E we described the method for calculating the snow force and torque acting on each element based on the orientation and depth of the snowboard in the snow. While the snow force was originally calculated at the mid-element, it is then transferred to the nodes (i.e. half the snow force is applied at each element node) as shown in (Figure F.15). The total snow force at each node is the sum of the forces of the adjacent elements. Here friction is neglected so the forces only act in the local *z* direction at the node. Thus the snow forces in the local *x-y-z* coordinates at the *i*th node are calculated

$$\begin{Bmatrix} f_i^{sx} \\ f_i^{sy} \\ f_i^{sz} \end{Bmatrix} = \tfrac{1}{2}\left([T_{LG}]_i^{node}\right)^{-1}[T_{LG}]_{i-1}^{ele}\begin{Bmatrix} 0 \\ 0 \\ f_{i-1}^{snow} \end{Bmatrix} + \tfrac{1}{2}\left([T_{LG}]_i^{node}\right)^{-1}[T_{LG}]_i^{ele}\begin{Bmatrix} 0 \\ 0 \\ f_i^{snow} \end{Bmatrix} \tag{F.27}$$

where $[T_{LG}]_i^{node}$ and $[T_{LG}]_i^{ele}$ are the coordinate transformations (Eqs. F.25 and F.26) relating the local $x-y-z$ and global $p-q-r$ coordinate systems. Here we neglect friction and define the snow force acting at the *i*th node as

$$f_i^s = \begin{Bmatrix} 0 \\ 0 \\ f_i^{sz} \end{Bmatrix} \tag{F.28}$$

The torque associated with the snow force is still assumed to act at the mid-segment, but for simplicity we designate the snow torque $t_i^s \left(= t_i^{snow}\right)$. The snow forces and torques are illustrated in Figure F.16.

F.5 The Solution Method

In this section we determine the deformed shape of the snowboard, the snow forces distributed along the base, and the boot forces given the position of the boots with respect to the snow surface.

The position of the boots is characterized by the location of point *B* with respect to the snow surface δ, the pitch angle η, and the roll angle β, and the yaw angle α (Figure F.17). (The yaw angle describes the position of the snowboard relative to the path. The snow models we consider (Brown and Outwater [7], elastic foundation) and, consequently, the deformed shape are independent of the yaw angle.) We calculate the forces in the snowboard assuming the two boots support the snowboard in such a way that the system is statically determinate. The resultant of the snow forces and the boot forces are shown in Figure F.18. The top illustration shows three forces and three moments acting at each boot, meaning that the snowboard is six times statically indeterminate. Here we assume (Figure F.18, bottom) that $\hat{M}^x$ and $\hat{M}^z$ are zero at both boots $\left(\hat{M}_{nf}^{x} = \hat{M}_{nb}^{x} = \hat{M}_{nf}^{z} = \hat{M}_{nb}^{z} = 0\right)$, while the forces F^{y} and torques T^{y} have identical values at each boot $\left(F_{nf}^{y} = F_{nb}^{y} \text{ and } T_{nf}^{y} = T_{nb}^{y}\right)$. Hence, the boot forces can be described by six parameters which can be uniquely determined if the snow forces along the snowboard are given.

The solution is highly non-linear for two reasons:

- the contact area depends on the applied load
- the deformation of the snowboard is not negligible

We employ the incremental method with load correction [5, page 505] to determine the relationship between the deformed shape, the snow forces, and the boot forces. Accordingly, we prescribe values of pitch η, roll β, and yaw α (it should be noted, however, that the value of yaw α does not influence our calculations) and displace the snowboard by pushing point *B* into the snow in small $\Delta\delta$ increments (Figure F.19). The snowboard is initially undeformed and the forward and aft contact points are resting on the surface of the snow. Because of the camber of the snowboard and the roll angle, point *B* may be above the surface of the snow. As the snowboard is displaced into the snow we define the distance Ξ as the displacement of point *B* from the initial undeformed position where Ξ is related to δ as shown in Figure F.19. The details of the incremental method with load correction are given in [5, page 505]. We recall that the incremental method involves two tasks:

- the calculation of the tangent stiffness matrix
- the calculation of the equilibrating loads given a deformed shape the (i.e. we must determine the forces and torques which would cause the prescribed deformed shape)

To begin we consider the second task because we will need to calculate the equilibrating forces and torques in order to form the tangent stiffness matrix.

Equilibrating Forces

Previously, we identified those forces which act on the snowboard:

- the snow forces f_i^s at each node (i = 1, 2,…, n)
- the snow torques t_i^s at each element (i = 1, 2,…, n-1)
- boot forces acting at the two designated elements *nb* and *nf*, which contain six unknowns F_{nb}^x, F_{nf}^x, F^y, F_{nb}^z, F_{nf}^z, T^y (Figure F.18).

For a prescribed deformed snowboard shape and given orientation (i.e. specified values of pitch η, roll β, and depth of point B δ) the snow forces can be uniquely calculated. However, when applied these forces may result in a different deformed shape than the one prescribed. Therefore, in addition to the snow forces, we apply "equilibrating" forces at each node f_i^d and equilibrating torques at each element t_i^d (Figure F.20). Thus, the total force at each node and the total torque at each element are

$$f_i^e = f_i^s + f_i^d \qquad t_i^e = t_i^s + t_i^d \tag{F.29}$$

The value of these equilibrating forces and torques are chosen such that the application of the local forces and torques results in the prescribed shape. Note that at nodes *nb* and *nf* the equilibrating forces are equal to the boot forces $f_{nb}^d = F_{nb}^z$ and $f_{nf}^d = F_{nf}^z$, while at elements *nb* and *nf*-1 the equilibrating torques equal the boot torques $t_{nb}^d = T_{nb}^y = T^y$ and $t_{nf-1}^d = T_{nf-1}^y = T^y$. The internal moments and torques acting at the nodes are related to the angles θ^b and γ^t by the stiffnesses defined in Eqs. F.6 and F.7. In matrix form the relationship between the internal moments and torques and the bending and twist angles may be expressed as

$$\begin{Bmatrix} \hat{M}_2^x \\ \vdots \\ \hat{M}_{n-1}^x \\ \hat{T}_2 \\ \vdots \\ \hat{T}_{n-1} \end{Bmatrix} = \begin{bmatrix} k_2^{EI} & & & k_2^C & & \\ & \ddots & & & \ddots & \\ & & k_{n-1}^{EI} & & & k_{n-1}^C \\ k_2^C & & & k_2^{GJ} & & \\ & \ddots & & & \ddots & \\ & & k_{n-1}^C & & & k_{n-1}^{GJ} \end{bmatrix}^{-1} \begin{Bmatrix} \theta_2^b \\ \vdots \\ \theta_{n-1}^b \\ \gamma_2^t \\ \vdots \\ \gamma_{n-1}^t \end{Bmatrix} \tag{F.30}$$

Thus, the internal moments $\hat{M}_i^x$ and torques $\hat{T}_i$ can be calculated because the bending and twist angles θ^b and γ^t, as well as the spring stiffnesses k_i^{EI}, k_i^{GJ} and k_i^C are known.

We summarize below the 2*n*+2 unknowns which must be calculated

- equilibrium forces at each node $f_i^e \quad i = 1, 2, \ldots, n$ (*n* unknowns)
- equilibrium torques at each element $t_i^e \quad i = 1, 2, \ldots, n-1$ (*n*-1 unknowns)

  (Knowing the equilibrium forces f_i^e and torques t_i^e as well as the snow forces f_i^s and torques t_i^s, the equilibrating forces f_i^d and torques t_i^d may be calculated using Eq. F.29. Thus, the boot forces $F_{nb}^z = f_{nb}^d$, $F_{nf}^z = f_{nf}^d$, and boot torques $T_{nb}^y = t_{nb}^d = T^y$, $T_{nf-1}^y = t_{nf-1}^d = T^y$ are known.)
- the boot forces F_{nb}^x, F_{nf}^x and $F_{nf}^y = F_{nb}^y = F^y$ (3 unknowns)

The following equilibrium equations must be satisfied to determine the above unknowns

- moment equilibrium at node *i* (Figure F.21, top) working from the tail *(i* = 2, 3, … , *n*-1)
- torque equilibrium at node *i* (Figure F.21, bottom) working from the tail *(* *i*= 2, 3, … , *n*-1)
- six global equilibrium equations

We simplify the solution by neglecting the minor contributions of the boot forces F_{nb}^x, F_{nf}^x, and F^y in the moments $\hat{M}_i^x$ and torques $\hat{T}_i$. In this case, the following 2*n*-1 equations are needed to calculate the unknowns f_i^e and t_i^e. Moment equilibrium at node *i* (working from the tail of the snowboard) may be expressed as