

Figure A.3: The roll  $\beta$ , pitch  $\eta$ , yaw  $\alpha$  angles. The angles shown are positive.

# Appendix B

## Centroid Location

In this section the centroid location is determined for a snowboard cross section with an arbitrary layup. The centroid,  $C$ , is located at such a position that the cross section's axis remains straight when an axial force is applied at the centroid. However, when the layup is unbalanced an applied axial load may cause the snowboard to twist [4].

The following expression relates the curvatures of the axis running through the origin  $D$  of the  $\bar{x} - \bar{y} - \bar{z}$  coordinate system to an axial load applied at the centroid (Figure B.1)

$$\begin{Bmatrix} \frac{1}{\rho_{\bar{x}}} \\ \frac{1}{\rho_{\bar{z}}} \end{Bmatrix} = \begin{bmatrix} \bar{W}_{12} & \bar{W}_{22} & \bar{W}_{23} \\ \bar{W}_{13} & \bar{W}_{23} & \bar{W}_{33} \end{bmatrix} \begin{Bmatrix} 1 \\ z_C \\ x_C \end{Bmatrix} \hat{N}_{\bar{y}} \quad (\text{B.1})$$

If the  $\bar{x} - \bar{y} - \bar{z}$  and  $x - y - z$  coordinate systems are coincident and  $\hat{N}_{\bar{y}}$  is applied at  $C$ , the curvatures of the axis passing through the centroid are zero thus

$$\frac{1}{\rho_x} = \frac{1}{\rho_z} = \frac{1}{\rho_{\bar{x}}} = \frac{1}{\rho_{\bar{z}}} = 0 \quad (\text{B.2})$$

Combining these equations, the position of the centroid may be determined

$$\begin{Bmatrix} z_C \\ x_C \end{Bmatrix} = - \begin{bmatrix} \bar{W}_{22} & \bar{W}_{23} \\ \bar{W}_{23} & \bar{W}_{33} \end{bmatrix}^{-1} \begin{bmatrix} \bar{W}_{12} \\ \bar{W}_{13} \end{bmatrix} \quad (\text{B.3})$$

where  $[\bar{W}]$  is the compliance matrix defined in Appendix C.

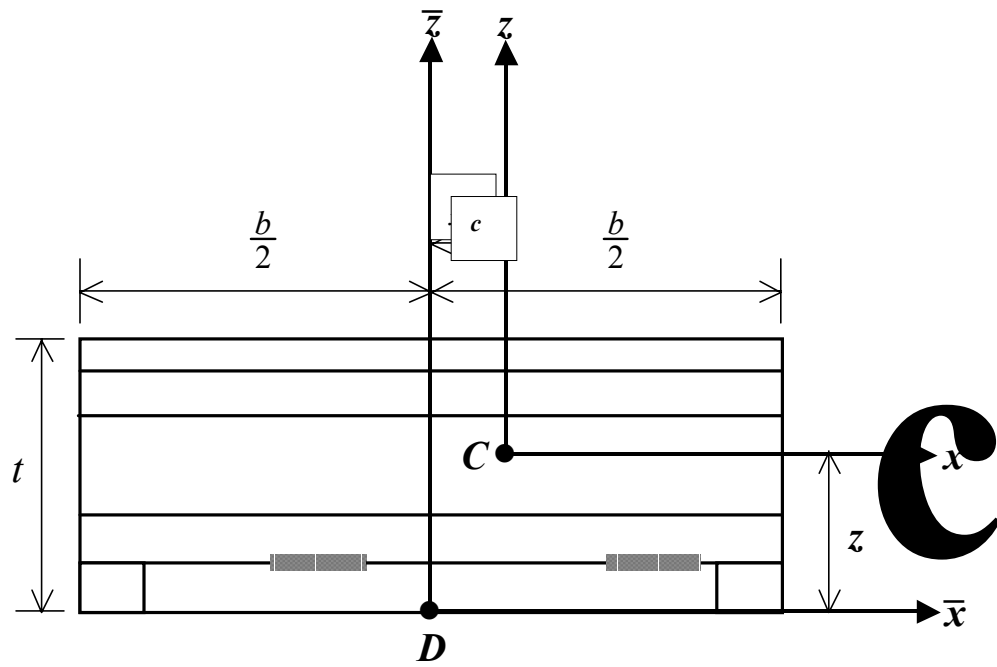


Figure B.1: The centroid location of a snowboard cross section.

## Appendix C

### Compliance Matrix

Composite theory may be applied to determine the relationship between the in-plane forces and moments and the strains and curvatures for the laminated snowboard. Here we consider two coordinate systems. The  $x - y - z$  coordinate system is attached to the centroid while the  $\bar{x} - \bar{y} - \bar{z}$  coordinate system is aligned with the mid-plane of the cross section (Figure B.1). As for the ski, we consider the three in-plane moments and three in-plane forces acting on a laminate as depicted in (Figure C.1). The in-plane forces and moments are related to the strains and curvatures by the compliance matrix  $[\bar{W}]$  and stiffness matrix  $[\bar{P}]$  in the  $\bar{x} - \bar{y} - \bar{z}$  coordinate system [4, page 267]

$$\begin{pmatrix} \varepsilon_y^0 \\ \frac{1}{\rho_{\bar{x}}} \\ \frac{1}{\rho_{\bar{z}}} \\ \bar{\mathfrak{g}} \end{pmatrix} = \begin{bmatrix} \bar{W}_{11} & \bar{W}_{12} & \bar{W}_{13} & \bar{W}_{14} \\ \bar{W}_{12} & \bar{W}_{22} & \bar{W}_{23} & \bar{W}_{24} \\ \bar{W}_{13} & \bar{W}_{23} & \bar{W}_{33} & \bar{W}_{34} \\ \bar{W}_{14} & \bar{W}_{24} & \bar{W}_{34} & \bar{W}_{44} \end{bmatrix} \begin{pmatrix} \hat{N} \\ \hat{M}_{\bar{x}} \\ \hat{M}_{\bar{z}} \\ \hat{T} \end{pmatrix}$$

$$\begin{pmatrix} \hat{N} \\ \hat{M}_{\bar{x}} \\ \hat{M}_{\bar{z}} \\ \hat{T}_y \end{pmatrix} = \begin{bmatrix} \bar{P}_{11} & \bar{P}_{12} & \bar{P}_{13} & \bar{P}_{14} \\ \bar{P}_{12} & \bar{P}_{22} & \bar{P}_{23} & \bar{P}_{24} \\ \bar{P}_{13} & \bar{P}_{23} & \bar{P}_{33} & \bar{P}_{34} \\ \bar{P}_{14} & \bar{P}_{24} & \bar{P}_{34} & \bar{P}_{44} \end{bmatrix} \begin{pmatrix} \varepsilon_y^0 \\ \frac{1}{\rho_{\bar{x}}} \\ \frac{1}{\rho_{\bar{z}}} \\ \bar{\mathfrak{g}} \end{pmatrix}$$

(C.1)

while in the  $x - y - z$  coordinate system [4, page 209]

$$\begin{pmatrix} \varepsilon_y^0 \\ \frac{1}{\rho_x} \\ \frac{1}{\rho_z} \\ \bar{\mathfrak{g}} \end{pmatrix} = \begin{bmatrix} W_{11} & W_{12} & W_{13} & W_{14} \\ W_{12} & W_{22} & W_{23} & W_{24} \\ W_{13} & W_{23} & W_{33} & W_{34} \\ W_{14} & W_{24} & W_{34} & W_{44} \end{bmatrix} \begin{pmatrix} \hat{N} \\ \hat{M}_x \\ \hat{M}_z \\ \hat{T} \end{pmatrix}$$

$$\begin{pmatrix} \hat{N} \\ \hat{M}_x \\ \hat{M}_z \\ \hat{T}_y \end{pmatrix} = \begin{bmatrix} P_{11} & P_{12} & P_{13} & P_{14} \\ P_{12} & P_{22} & P_{23} & P_{24} \\ P_{13} & P_{23} & P_{33} & P_{34} \\ P_{14} & P_{24} & P_{34} & P_{44} \end{bmatrix} \begin{pmatrix} \varepsilon_y^0 \\ \frac{1}{\rho_x} \\ \frac{1}{\rho_z} \\ \bar{\mathfrak{g}} \end{pmatrix}$$

(C.2)

where the forces and moments (Figure C.1) are related to the force per unit length and moment per unit length (Figure C.2) by the following equations

$$\begin{aligned} N_y &= \frac{\hat{N}}{b} \\ M_y &= -\frac{\hat{M}_x}{b} \\ \hat{T} &= -2bM_{xy} \end{aligned} \quad (C.3)$$

The compliance and stiffness matrices are related by

$$[\bar{W}] = [\bar{P}]^{-1} \quad (C.4)$$

First we determine the stiffness matrix in the  $\bar{x} - \bar{y} - \bar{z}$  coordinate system. The snowboard cross section is comprised of  $n_G$  different groups (Figure C.3) all of which contribute to the stiffness of the board. Therefore the following expression [4, page 268] may be used to calculate the stiffness matrix of the complete cross section

$$[\bar{P}] = \sum_{j=1}^{n_G} [R_j]^T [\omega_j]^{-1} [R_j] \quad (C.5)$$

where  $[R_j]$  may be expressed as with  $\bar{x}_j$  and  $\bar{z}_j$  defined in (Figure C.4)

$$[R_j] = \begin{bmatrix} 1 & \bar{z}_j & \bar{x}_j & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (C.6)$$

where  $\bar{x}_j$  and  $\bar{z}_j$  defined in (Figure C.4). The  $[\omega_j]$  matrix is defined as

$$[\omega_j] = \frac{1}{b_j} \begin{bmatrix} \alpha_{22} & \beta_{22} & 0 & -\frac{1}{2}\beta_{26} \\ \beta_{22} & \delta_{22} & 0 & -\frac{1}{2}\delta_{26} \\ 0 & 0 & \frac{12}{(\tilde{A}_{11})_j b_j^2} & 0 \\ -\frac{1}{2}\beta_{26} & -\frac{1}{2}\delta_{26} & 0 & \frac{1}{4}\delta_{66} \end{bmatrix} \quad (C.7)$$

where  $\tilde{A}_{11}$  is the 1,1 element of the  $[\tilde{A}]$  matrix given as

$$\begin{bmatrix} \tilde{A}_{11} & \tilde{A}_{12} & \tilde{A}_{13} \\ \tilde{A}_{12} & \tilde{A}_{22} & \tilde{A}_{23} \\ \tilde{A}_{13} & \tilde{A}_{23} & \tilde{A}_{33} \end{bmatrix} = \begin{bmatrix} \alpha_{22} & \beta_{22} & \beta_{26} \\ \beta_{22} & \delta_{22} & \delta_{26} \\ \beta_{26} & \delta_{26} & \delta_{66} \end{bmatrix} \quad (\text{C.8})$$

In the equations above  $\alpha$ ,  $\beta$ , and  $\delta$  are the elements of the inverse of the  $A$ - $B$ - $D$  stiffness matrix

$$\begin{bmatrix} \alpha & \beta \\ \beta^T & \delta \end{bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix}^{-1} \quad (\text{C.9})$$

The stiffness  $[\bar{P}]$  and compliance  $[\bar{W}]$  matrices in the  $\bar{x}-\bar{y}-\bar{z}$  coordinate system are related to those in the  $x-y-z$  centroid coordinate system  $[P], [W]$  by the following relationship [4, page 268]

$$\begin{aligned} [W] &= [R_b]^T [\bar{W}] [R_b] \\ [P] &= [W]^{-1} \end{aligned} \quad (\text{C.10})$$

In this case,  $[R_b]$  may be determined by the following expression

$$[R_b] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ z_C & 1 & 0 & 0 \\ x_C & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (\text{C.11})$$

where  $x_C$  and  $x_C$  are the coordinates of the centroid (Figure C.4).

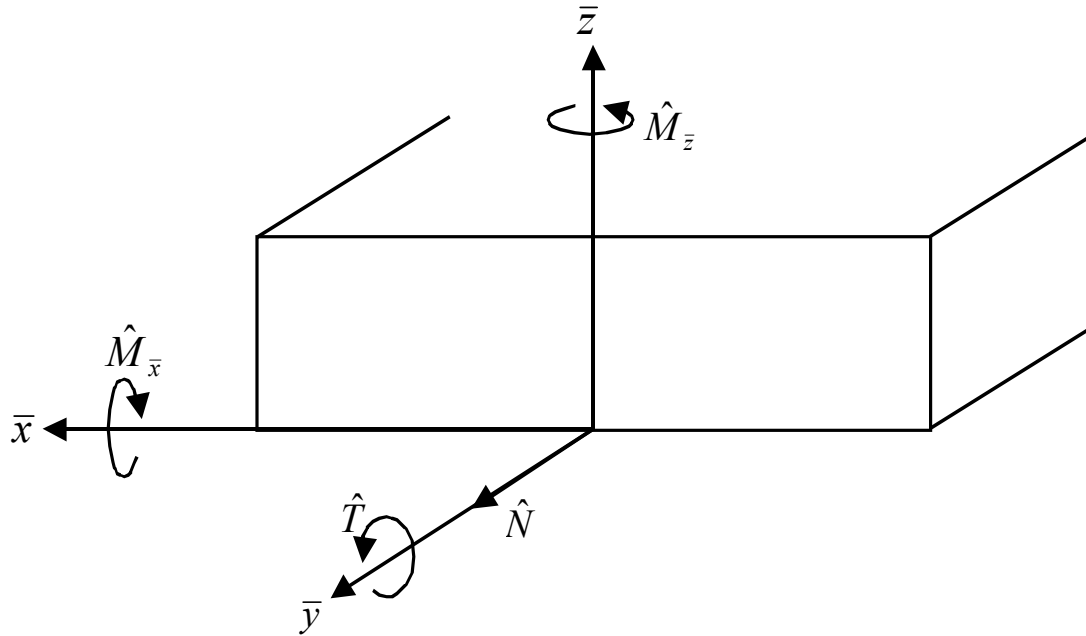


Figure C.1: The normal force  $\hat{N}$ , the bending moments  $\hat{M}_{\bar{x}}$ ,  $\hat{M}_{\bar{z}}$ , and the torque  $\hat{T}$  inside a beam in the  $\bar{x} - \bar{y} - \bar{z}$  coordinate system.

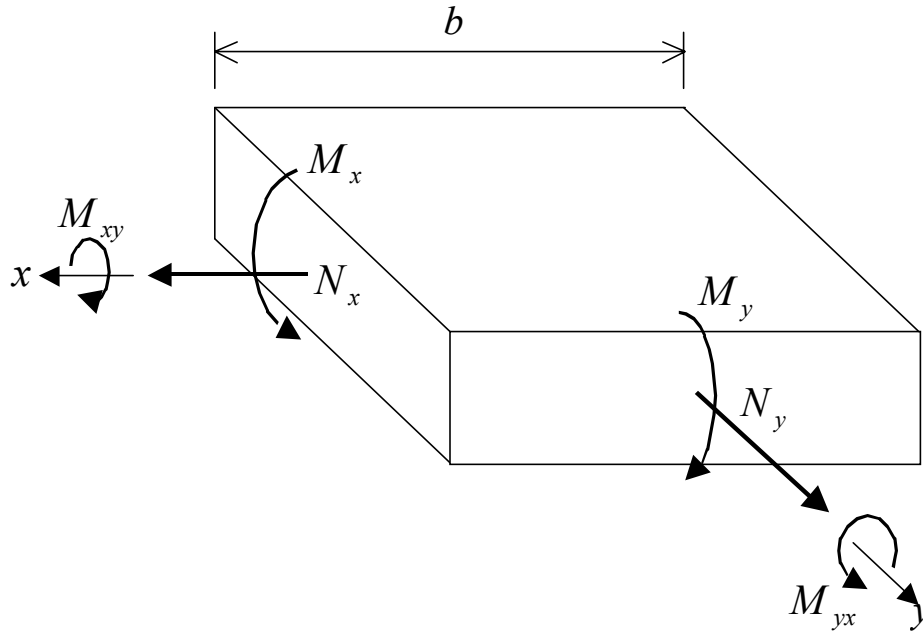


Figure C.2: The in-plane forces per unit length and moments per unit length acting on a segment.

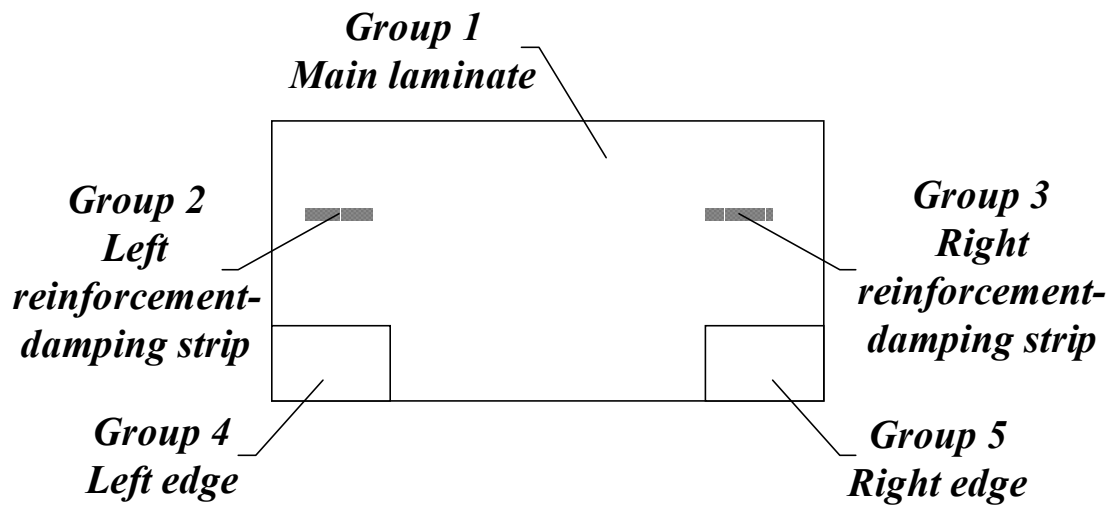


Figure C.3: Group designation of a snowboard cross section.

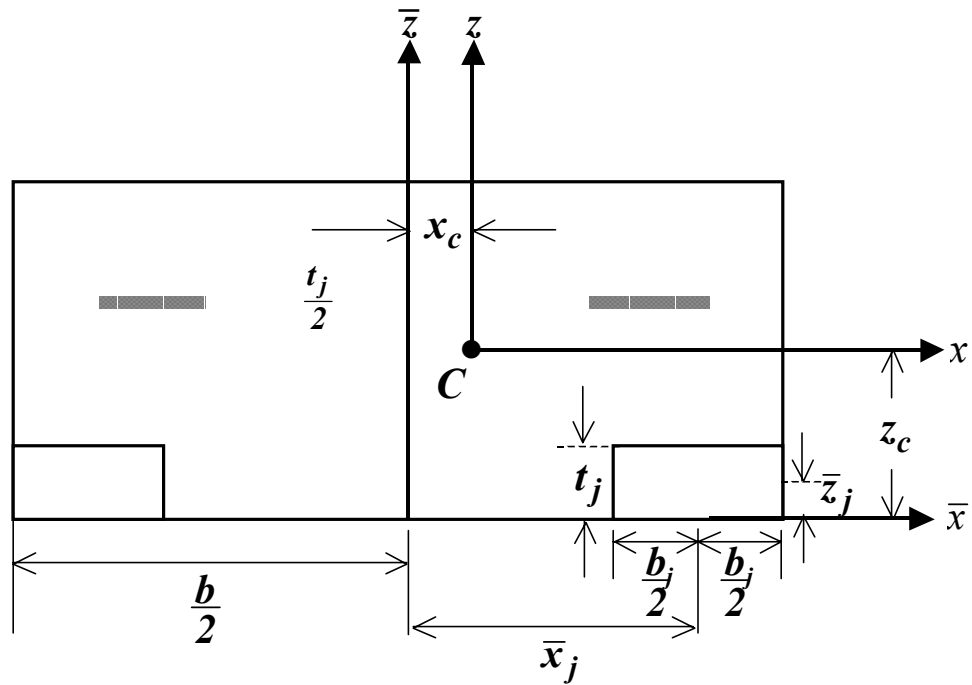


Figure C.4: The position of group  $j$  relative to the  $\bar{x} - \bar{y} - \bar{z}$  coordinate system. The thickness of the ply group is  $t_j$ , the width is  $b_j$ , the distance from the  $\bar{x} - \bar{y} - \bar{z}$  origin to the mid-width of the  $j$ th group is  $\bar{x}_j$ , the distance from the origin to the mid-height of the  $j$ th group is  $\bar{z}_j$ . The position of the centroid relative to the  $\bar{x} - \bar{y} - \bar{z}$  coordinate system is indicated by  $C$  and the distances  $x_c$  and  $z_c$ .

# Appendix D

## Transverse Inertia Force Calculations

In Chapter 4 we stated that as a snowboarder negotiates a course, his center of gravity position oscillates. We illustrated the movement of the center of gravity position and defined the parameters used to calculate this position at any point along the path. Here we will determine the acceleration of the center of gravity position due to this motion of the snowboarder which will allow us to define the transverse inertia force.

Here we consider one turn (Figure D.1), thus

$$h_{cg} = c_1 H_{sb} + \frac{1}{2} c_2 c_3 \frac{v}{v_{\max}} H_{sb} \cos\left(\frac{2\pi}{R\Omega} s\right) \quad (\text{D.1})$$

where  $h_{cg}$  is the height of the center of gravity above the snowboard,  $c_1$ ,  $c_2$ ,  $c_3$  are constants that are based on the snowboarder's height and ability,  $H_{sb}$  is the height of the snowboarder,  $v$  is the speed,  $v_{\max}$  is the maximum speed the snowboarder can maintain,  $s$  is the position along the path, and  $R\Omega$  is the length of one turn.

To approach this problem, we break the turn into  $n$  segments each of length  $\Delta s_i$  (Figure D.1). We calculate the position of the center of gravity  $(h_{cg})_i$  at every point along the path. When the location of the center of gravity position is known at points  $i-1$ ,  $i$ , and  $i+1$  are known the acceleration of the center of gravity due to the motion of the snowboarder relative to the snowboard is approximated by the central difference method [12, page 186] as follows

$$(a_{cg})_i = \frac{(h_{cg})_{i-1} - 2(h_{cg})_i + (h_{cg})_{i+1}}{\left(\frac{\Delta t_{i+1} + \Delta t_i}{2}\right)^2} \quad (\text{D.2})$$

To calculate  $(a_{cg})_0$  we need to know the value of  $(h_{cg})_{i-1}$  which we approximate as  $(h_{cg})_1$ . We further assume in the calculation of  $(h_{cg})_{i+1}$  that  $(v_{cg})_{i+1} = (v_{cg})_i$ . We can determine the time  $\Delta t_i$  it takes to travel the distance  $\Delta s_i$ .

$$\Delta t_i = \frac{\Delta s_i}{v_{i-1}} \quad (\text{D.3})$$

Thus, the transverse inertia force is

$$(T)_i = m_{sb}(a_{cg})_i \quad (\text{D.4})$$

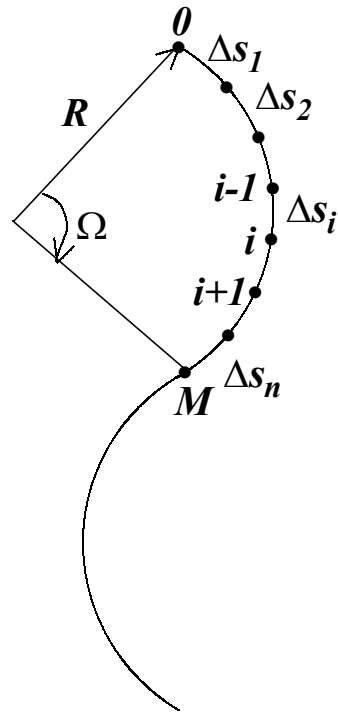


Figure D.1: Segmentation of the path.

# Appendix E

## Elastic Foundation Snow Forces

The Brown and Outwater [7] snow model yields infinite snow forces as the roll angle approaches zero, therefore we present an alternative model for defining the snow forces, the elastic foundation model.

We consider the snowboard to be supported by a series of springs along the base, each of stiffness  $K_{sn}$  as shown in Figure 4.7. Thus, the snow force per unit area may be expressed as

$$\sigma_f = K_{sn} \frac{\Delta u}{U} \quad (\text{E.1})$$

where  $\Delta u$  is the distance the snowboard moved into the snow,  $U$  is the depth of the snow, and  $K_{sn}$  is the stiffness of the snow. We take  $K_{sn}$  to be constant for a given snow consistency. When the snowboard is at a prescribed roll angle we can determine the acting snow force. In Figure E.1 we define the geometry used to determine the snow force. The following relationships hold true

$$u' + u'' = \frac{U}{\cos \beta} \quad u' + u''' = \frac{u_{\max}}{\cos \beta} \quad b' = \frac{u_{\max}}{\sin \beta} \quad u''' = (b' - b'') \tan \beta \quad (\text{E.2})$$

Thus, the snow force per unit length may be expressed as

$$\begin{aligned} (f^{snow})_{EF} &= \int_0^{b'} \sigma_f db'' = \int_0^{b'} K_{sn} \frac{u'}{u' + u''} db'' \\ &= \int_0^{b'} \frac{K_{sn}}{U} \cos \beta \left[ \frac{u_{\max}}{\cos \beta} - (b' - b'') \tan \beta \right] db'' \\ &= \frac{K_{sn}}{U} \left( u_{\max} b' - \frac{1}{2} b'^2 \sin \beta \right) \end{aligned} \quad (\text{E.3})$$