

# MODELING THE MECHANICAL CHARACTERISTICS AND ON-SNOW PERFORMANCE OF SNOWBOARDS

A DISSERTATION

SUBMITTED TO THE DEPARTMENT OF AERONAUTICS AND ASTRONAUTICS

AND THE COMMITTEE ON GRADUATE STUDIES

OF STANFORD UNIVERSITY

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS

FOR THE DEGREE OF

DOCTOR OF PHILOSOPHY

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March 2003

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# Signatures

# Abstract

Models were developed for calculating the mechanical characteristics and the on-snow performance of snowboards. The snowboards are constructed of layers of materials which may include wood, foam, honeycomb, fiber reinforced composites, and polymeric materials. The models pertaining to the mechanical characteristics provide the bending and torsional stiffnesses, the flex, and the twist. A computer code “Snowboard-MECH” was written which yields numerical values of these characteristics. The model pertaining to the on-snow performance simulates the travel of a snowboarder of given height, weight, and skill level down an S-shaped course. A computer code “Snowboard-TURN” was written in support of this model. This code calculates the time it takes the snowboarder to complete the course. The two computer codes were verified by comparing the outputs of these codes to laboratory data and to data generated by a snowboarder completing a prescribed S-shaped course. The results generated by the models and the data are in agreement, lending confidence to the models and the corresponding computer codes. A procedure is described by which the computer codes developed in this study can be utilized in the design of snowboards.

# Acknowledgements

My Stanford experience has been an interesting adventure highlighted by unexpected twists, good fortune, and a colorful cast of characters. To the many who played important roles as advisors, supporters, cohorts, confidants, and welcome distractions I am forever indebted, for without your influence I might never have escaped that dungeon of Durand. Instead, I can offer dynamic tales full of intrigue and accomplishment, complete with humorous anecdotes, with only minor embellishment.

I am grateful to the Herbert Kunzel Foundation for funding my Stanford Graduate Fellowship. I would like to thank Cameron Andrus and K2 Corporation for providing snowboards and engineering data for verification of the computer models. I would also like to recognize Alpine Meadows Ski Resort and for their cooperation with our on-snow testing. My thanks to Guttorm Opshaug for his help with the GPS system and his participation in the field tests. And to my friends who teamed up for problem sets, prepped me for quals, answered my calls from Hungary, and ensured the Pony was never neglected—my sincerest appreciation.

Dr. Alison Nordt is a skilled engineer, an enthusiastic friend, and one of the most accommodating people I have ever met. It was her ski design project that brought me to Stanford, and it was her commitment and dedication that helped me to earn my degree. No call for help went unanswered and no silly misunderstanding was ever dismissed. I suspect she will be most grateful to celebrate my graduation because she will no longer be burdened with e-mail entitled “Code question.”

I consider myself extremely privileged to have had the opportunity to work with Prof. George Springer and Prof. László Kollár. They are brilliant men, each respected for his intelligence and contribution to science, but they are more distinguished by their character and integrity. I recognize that I am lucky—very few students have an opportunity to work with such dedicated advisors. Despite my greatest effort, I never managed to arise to contemplate my thesis before Prof. Springer, and while I tried often and repeatedly to impress László with my ignorance, I never managed to exhaust his patience. Perhaps I am most grateful for their faith and their expectation of success. I must also thank their families who excused my imposition and welcomed me into their homes. Hungarian hospitality puts southern hospitality to shame.

Alas, I've saved the best for last for it is my family to whom I am most indebted. In my lifetime they have endured countless tortures (4 o'clock ball games, music lessons gone awry, ballet "performances", the furrowed brow, mounting tuition) but they have always supported me. I seek them for advice, respect them for their decisions, and admire them for their conviction. I appreciate their compassion, their dedication, and their wit. They are most generous and self-sacrificing. They have given me every opportunity and I will always strive to make them proud. I am honored to know them, hope to be more like them, and love them very much.

# Contents

<b>Abstract</b>	<b>iv</b>
<b>Acknowledgements</b>	<b>v</b>
<b>Chapter 1 Introduction</b>	<b>1</b>
<b>Chapter 2 Problem Statement</b>	<b>3</b>
<b>Chapter 3 Bending and Torsion</b>	<b>12</b>
3.1 Computer Code “Snowboard-MECH”	13
3.2 Validation of Mechanical Characteristics	14
<b>Chapter 4 The Turn Model</b>	<b>28</b>
4.1 The Forces	28
4.2 Snow Model	31
4.3 Method of Solution	32
<b>Chapter 5 Validation: On-snow Tests</b>	<b>54</b>
<b>Chapter 6 Design Procedure: Concluding Remarks</b>	<b>65</b>
<b>Appendix A Pitch, Yaw, and Roll</b>	<b>67</b>
<b>Appendix B Centroid Location</b>	<b>72</b>
<b>Appendix C Compliance Matrix</b>	<b>74</b>
<b>Appendix D Transverse Inertia Force Calculations</b>	<b>82</b>
<b>Appendix E Elastic Foundation Snow Model</b>	<b>85</b>
<b>Appendix F Turn Model Calculations</b>	<b>89</b>
<b>Bibliography</b>	<b>130</b>

# List of Tables

Table 3.1	Input and output of the computer code “Snowboard-MECH” .....	25
Table 3.2	Input and output of the computer code “Snowboard-MECH” .....	26
Table 4.1	The input needed to calculate the time required to complete a specified course. ....	53
Table 5.1	Input to the K2 Spitfire 164 and K2 Astar 147 models. ....	63
Table 5.2	Input to the K2 Spitfire 164 and K2 Astar 147 models. ....	64

# List of Figures

Figure 2.1 Snowboard geometry with length $L$ , running length $L_r$ , width $b$ , thickness $t$ , camber $c$ , and offset.....	5
Figure 2.2 Snowboard construction.....	6
Figure 2.3 The weight distribution of the snowboarder; $W$ is the weight of the snowboarder, and $W^f$ and $W^b$ represent the weights on the front and back boots. ....	7
Figure 2.4 Description of the course; $R_1$ and $R_2$ are the radii of the two linked turns, and $R_1\Omega_1 + R_2\Omega_2$ is the length of the path. The snowboarder enters with speed $v_0$ , and $t_e$ is the time required to complete the course. ....	8
Figure 2.5 The motion of the center of gravity ‘up and down’ (left), ‘forward and back’ with respect to the normal $Z$ to the snowboard (middle), and ‘sideways’ with respect to the normal $q$ to the slope (right).....	9
Figure 2.6 Illustration of the pitch, roll, and yaw. Pitch and roll are about the snowboard’s transverse $X$ and longitudinal $Y$ axes respectively, and yaw is about the $q$ axis perpendicular to the snow. The $X$ , $Y$ , and $q$ coordinates are defined in Appendix A.....	10
Figure 2.7 Condition for executing a safe turn with ease.....	11
Figure 3.1 The bending moment and the curvature.....	15
Figure 3.2 Torque applied to the snowboard and the angle of twist. ....	16
Figure 3.3 The bend-twist coupling of a snowboard with unsymmetrical construction subjected to a bending moment.....	17
Figure 3.4 Snowboard in 3 point bending. $MRS$ is the mid-running surface, $FCP$ and $ACP$ are the forward and aft contact points. The flex is: $\text{Flex} = \frac{P}{\Delta}$ ..	18
Figure 3.5 Forebody and aftbody twists.....	19

Figure 3.6 Dimensions of the solid aluminum beam, the steel box beam, and the aluminum-wood-aluminum sandwich beam used in analysis. All dimensions are in meters.....	20
Figure 3.7 The layup of the K2 Astar 147 and the K2 Spitfire 164 snowboards. The top layer is 22 oz glass fabric for the Astar and 25 oz glass fabric for the Spitfire. The bottom layer is 22 oz glass fabric for both snowboards. The dimensions are given in Tables 3.1-3.3. ....	21
Figure 3.8 Bending stiffnesses of the K2 snowboards. ....	22
Figure 3.9 Flex calculated by the model compared to data.....	23
Figure 3.10 Twist calculated by the model compared to data. FCP and ACP indicate forebody and aftbody twist. ....	24
Figure 4.1 The path the snowboarder travels; $s$ is the distance along the path, $S_M$ is the distance to the inflection point between the two turns, and $S_E$ is the total distance along the path.....	38
Figure 4.2 The forces acting on the snowboarder and on the snowboard. $W$ is the weight, $C$ is the centrifugal force, $T$ is the transverse inertia force, $D_{air}$ is the air drag, $f_{snow}^n$ is the snow force normal to the base of the snowboard, and $f_{snow}^t$ is the friction along the base. ....	39
Figure 4.3 The distance between the center of gravity and the snowboard. At the “neutral” position of the center of gravity the distance is $h_{cg}^0$ . The maximum and minimum distances are $h_{cg}^0 + \frac{1}{2} \Delta h_{cg}$ and $h_{cg}^0 - \frac{1}{2} \Delta h_{cg}$ . ....	40
Figure 4.4 The center of gravity position along the path. ....	41
Figure 4.5 The boot forces and moments equilibrating the snow force (top); the boot forces and moments employed in the analysis (middle); and the “midpoint” forces and moments equilibrating the boot forces and moments (bottom). $L_B$ is the distance between the centers of the front and back boots.....	42
Figure 4.6 The position of the snowboard in the snow where $u_{max}$ is the depth of the deepest edge and $\phi$ is the rotation angle. ....	43

Figure 4.7 The elastic foundation model.....	44
Figure 4.8 The snowboard at a rotation angle $\phi$ .....	45
Figure 4.9 The snow force distribution along the base of the snowboard and the “midpoint” forces and moments in the $p - q - r$ coordinate system at a depth $\delta$ .....	46
Figure 4.10 Schematic of the “library” .....	47
Figure 4.11 The relevant data set. ....	48
Figure 4.12 The points in the relevant data set the do not satisfy the allowable ranges of roll $\beta$ and yaw $\alpha$ (points crossed out) and the $N - n_s$ viable “points” (open symbols). ....	49
Figure 4.13 The position of the center of gravity and the forces and moments at point $B$ .	
Figure 4.14 The process of elimination that yields the “midpoint” forces, “midpoint” moments, and shape at a specific point on the path pertaining to a specific snowboarder (weight distribution). ....	51
Figure 4.15 A segment of the path $\Delta s$ .....	52
Figure 5.1 The course traveled during the on-snow tests. The starting point for the K2 Spitfire 164 was at $\theta$ , and for the K2 Astar 147 at $A$ . ....	56
Figure 5.2 The on-snow test setup.....	57
Figure 5.3 The GPS system.....	58
Figure 5.4 The GPS position data versus the marked course.....	59
Figure 5.5 On-snow test results for the K2 Spitfire 164. The input to the model is in Tables 5.1-5.2.....	60
Figure 5.6 On-snow test results for the K2 Astar 147. The input to the model is in Tables 5.1-5.2.....	61
Figure 5.7 Time to complete the course: on-snow tests. The input to the models is in Tables 5.1-5.2.....	62

Figure 6.1	The preliminary design process. ....	66
Figure A.1	The $X$ - $Y$ - $Z$ coordinate system. ....	69
Figure A.2	The $p$ - $q$ - $r$ coordinate system which is attached to the path. ....	70
Figure A.3	The roll $\beta$ , pitch $\eta$ , yaw $\alpha$ angles. The angles shown are positive. ....	71
Figure B.1	The centroid location of a snowboard cross section. ....	73
Figure C.1	The normal force $\hat{N}$ , the bending moments $\hat{M}_{\bar{x}}$ , $\hat{M}_{\bar{z}}$ , and the torque $\hat{T}$ inside a beam in the $\bar{x}$ - $\bar{y}$ - $\bar{z}$ coordinate system. ....	78
Figure C.2	The in-plane forces per unit length and moments per unit length acting on a segment. ....	79
Figure C.3	Group designation of a snowboard cross section. ....	80
Figure C.4	The position of group $j$ relative to the $\bar{x}$ - $\bar{y}$ - $\bar{z}$ coordinate system. The thickness of the ply group is $t_j$ , the width is $b_j$ , the distance from the $\bar{x}$ - $\bar{y}$ - $\bar{z}$ origin to the mid-width of the $j$ th group is $\bar{x}_j$ , the distance from the origin to the mid-height of the $j$ th group is $\bar{z}_j$ . The position of the centroid relative to the $\bar{x}$ - $\bar{y}$ - $\bar{z}$ coordinate system is indicated by $C$ and the distances $x_c$ and $z_c$ . ....	81
Figure D.1	Segmentation of the path. ....	84
Figure E.1	The partially submerged snowboard. ....	87
Figure E.2	The snow force and torque. ....	88
Figure F.1	Segmentation of the snowboard. ....	107
Figure F.2	The $\tilde{X}$ - $\tilde{Y}$ - $\tilde{Z}$ coordinate system. $MRS$ is the mid-running surface, $FCP$ is the forward contact point, $ACP$ is the aft contact point, $L_r$ is the running length, and $b$ is the width. ....	108
Figure F.3	The position of the $i$ th node in the $\tilde{X}$ - $\tilde{Y}$ - $\tilde{Z}$ coordinate system. ....	109
Figure F.4	The offset angle $\lambda$ . ....	110

Figure F.5	The camber angle $\theta^c$ .....	111
Figure F.6	Representation of the snowboard by rigid bars connected at the nodes by springs of bending stiffness $k_i^{EI}$ , torsional stiffness $k_i^{GJ}$ , and coupling stiffness $k_i^c$ .....	112
Figure F.7	The points $a, b, c, d, e, f, g,$ and $h$ describing the $i$ th element. Point $a$ corresponds to node $i$ , point $c$ corresponds to node $i+1$ . Points $b, d, f, g,$ and $h$ define a surface representing the base of the element. Point $e$ represents the midpoint of the element and is located halfway between points $a$ and $c$ . .....	113
Figure F.8	The snowboard-fixed $X - Y - Z$ coordinate system.....	114
Figure F.9	Relationship between the $\tilde{X} - \tilde{Y} - \tilde{Z}$ and $X - Y - Z$ coordinate systems. Open circles represent the position of the mid-plane of the cross section. Black circles indicate the position of the centroid of the cross section. The $a-a$ line connects the centroids of the two boots. The point $B$ is located halfway between the two boots. The origin of the $\tilde{X} - \tilde{Y} - \tilde{Z}$ coordinate system is located at the $MRS$ at the midpoint between the two edges. The origin of the $X - Y - Z$ coordinate system is point $B$ . .....	115
Figure F.10	The total bending angle $\theta_i^t$ consisting of the camber angle $\theta_i^c$ and the bending angle $\theta_i^b$ .....	116
Figure F.11	The twist angle used to calculate snowboard shape.....	117
Figure F.12	Stage 1 in the determination of the snowboard position.....	118
Figure F.13	The orientation of the $Y$ axis with respect to the $p-q-r$ coordinate system. ....	119
Figure F.14	The angle $\Lambda$ between the $Z$ axis and the $q$ axis in the $q-r$ plane. ....	120
Figure F.15	The snow force applied at the $i$ th node. ....	121
Figure F.16	The snow forces acting at in the local $z$ direction and the nodes and the snow torques acting in the local $y$ direction at the mid-segment. ....	122

Figure F.17	The position of the snowboard in the snow. ....	123
Figure F.18	The boot forces applied to the snowboard at the two boots and the equivalent snow forces at point $B$ . ....	124
Figure F.19	The relationship between boot force $F_Z = (F_{nb}^z)_Z + (F_{nf}^z)_Z$ (snowboard-fixed $Z$ direction) and the depth of point $B$ relative to the snow surface	
	$(F_{nb}^z)_Z = \{0 \quad 0 \quad 1\} [T_{SG}]^{-1} [T_{LG}]_{nb}^{node} \begin{Bmatrix} 0 \\ 0 \\ F_{nb}^z \end{Bmatrix}$	
	(q direction), where	125
Figure F.20	The snow forces $f^s$ and torques $t^s$ and the equilibrating forces $f^d$ and torques $t^d$ acting on the snowboard. At the boot nodes $f_{nb}^d = F_{nb}^z, f_{nf}^d = F_{nf}^z$ , while at the boot elements $t_{nb}^d = t_{nf-1}^d = T^y$ .	126
Figure F.21	The internal moment $\hat{M}_i^x$ and torque $\hat{T}_i$ at node $i$ and the associated equilibrium forces $f^e$ and torques $t^e$ .	127
Figure F.22	The position vector $\hat{r}_{ij}$ between node $i$ and the base of the snowboard at node $j$ . ....	128
Figure F.23	The incremental displacement of the snowboard into the snow. The pitch $\eta$ , roll $\beta$ , and yaw $\alpha$ are prescribed while the depth $\delta$ of point $B$ is changed. ....	129

# List of Symbols

## Latin Letters

$ACP$	.....aft contact point
$A_F$	.....frontal area
$a$	.....distance used to calculate snow torque
$a - a$	.....line connecting the centers of the two boots
$a_p$	.....acceleration in the direction of the path ( $p$ direction)
$b$	.....width of the snowboard
$b_s$	.....width of the snowboard in contact with snow
$C$	.....centroid
$C$	.....centrifugal force
$C_D$	.....drag coefficient
$C_p, C_q, C_r$	.....components of centrifugal force in the path coordinate system
$c$	.....camber height
cg	.....center of gravity
$c_1, c_2, c_3$	.....center of gravity coefficients
$D$	.....total drag
$D_{air}$	.....air drag
$dL$	.....small fraction of the length of the snowboard
$dm$	.....mass of a segment of the snowboard
$E$	.....Young's modulus
$e$	.....direction in the plane of the hill perpendicular to the fall line
$(EI)'$	.....equivalent bending stiffness
$f$	.....spring force
$FCP$	.....forward contact point
$F_b^x, F_b^y, F_b^z$	.....forces applied at the back boot
$F_f^x, F_f^y, F_f^z$	.....forces applied at the front boot
$F_p^B, F_q^B, F_r^B$	....."midpoint" forces at point $B$ in the path coordinate system
$F^y$	.....axial force applied at each boot
$F_Z$	.....sum of boots forces in the $Z$ direction
$f$	.....direction of the fall line
$f^d$	.....equilibrating forces
$f^e$	.....equilibrium forces